Least-Squares Method Continued

-30th January 2004

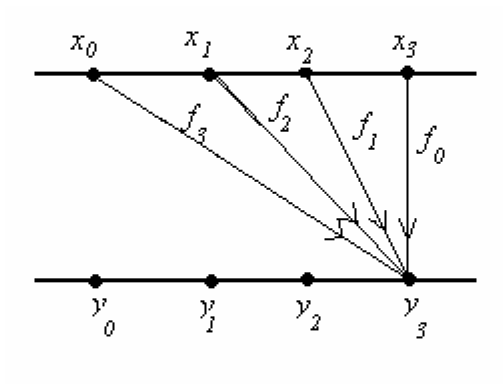
Why do we need Auto Regressive Systems?

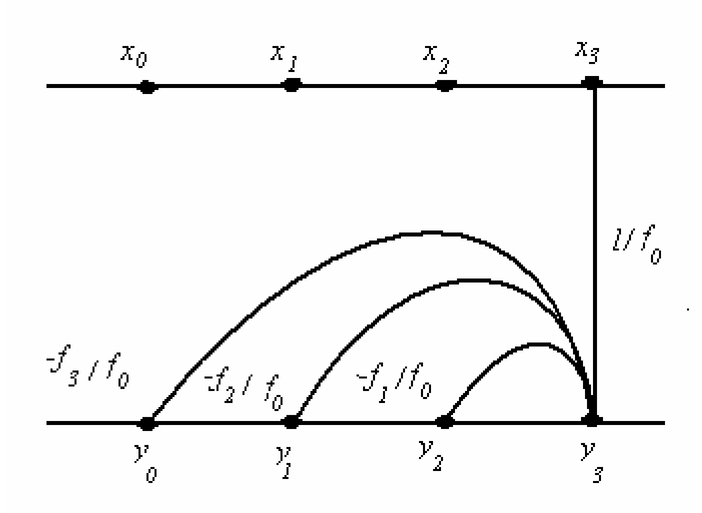
If the system plant is recursive e.g. standing wave echo.
Moving Average requires more latches
You need many coefficients for FIR
You can use it in integrator, differentiator and shock absorber

Modeling AR systems:

1. Model the inverse of the AR system as MA

We know that if the coefficients of the Moving Average system are 1, f_1,f_2 and f_3 , the coefficients of the (inverse) AR system are 1, -f_1,-f_2 and -f_3 respectively.





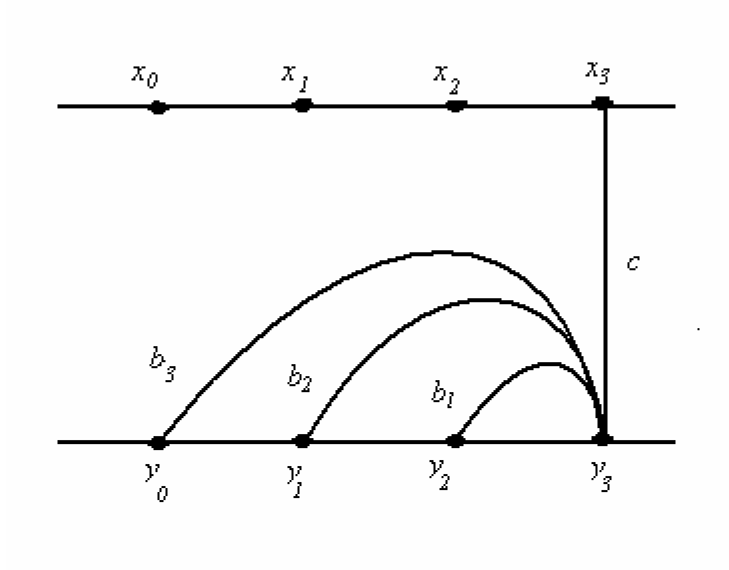
2. <u>Direct AR modeling</u>

We know the equation of the AR filter.

$y_0 = cx_0 + b_1y_{-1} + b_2y_{-2} + b_3y_{-3} \ldots\ldots$

Now since we start with zeroes, we can write the AR filter in matrix form as follows:

$$\begin{bmatrix} 0 & 0 & 0 & x_0 \\ y_0 & 0 & 0 & x_1 \\ y_0 & y_1 & 0 & x_2 \\ y_0 & y_1 & y_2 & x_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ c \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

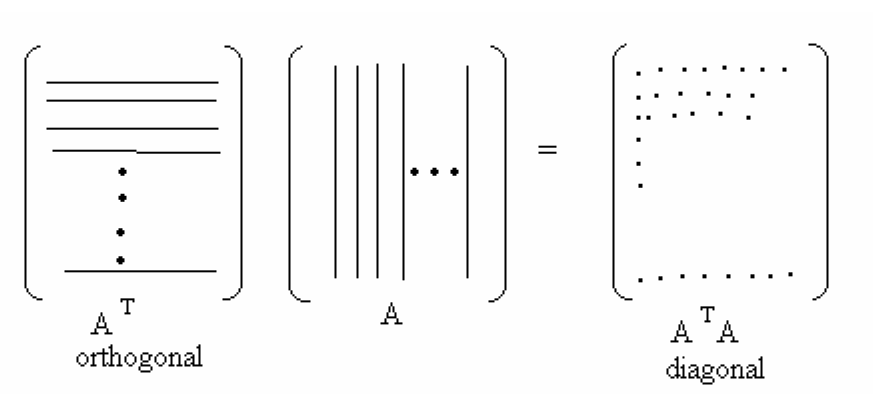


This method gives the same answer as the inverse. But the inverse method is easier.
Direct AR is good if we have c=0. (c=0 indicates that the system does not depend on any input)

Note: The matrix that we observe in the direct AR modeling is not the Toeplitz matrix, as it is not the shift in shift out x thing.

$A^T A$:
Some stuff about $A^T A$.
It is the dot product of the bases and is the skinny matrix whose size is the order of the system.

A^T has the bases on the rows and A on the columns.



If A^T is orthogonal then $A^T A$ is diagonal. Impulse is an example of orthogonal bases.

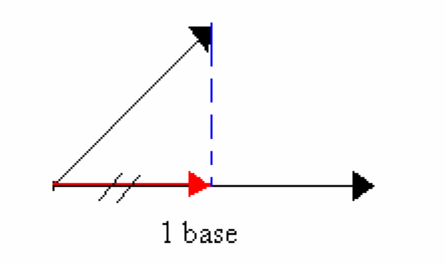
$A^T A$ is the auto correlation of the original matrix and it is a square, symmetric matrix and the dot product of the bases.

Its dimension = no. of coefficients of the system.

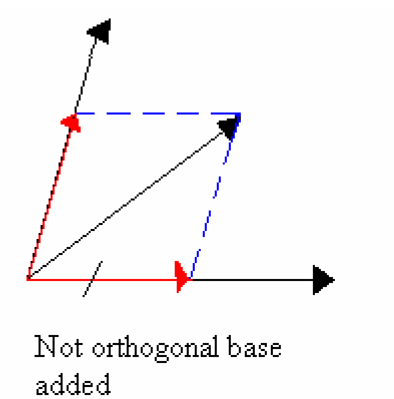
<u>Advantage of orthogonal bases.</u>

If the bases are not orthogonal, then adding a base changes how the previous filter is used.

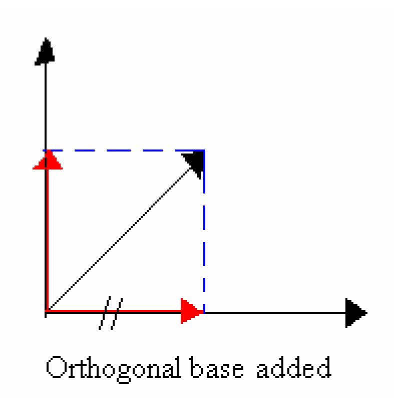
If there is only one base, so the red line shows how the filter is used



Now the new base is not orthogonal, we wont know how to add the previous filter, its not equal to the one base case.



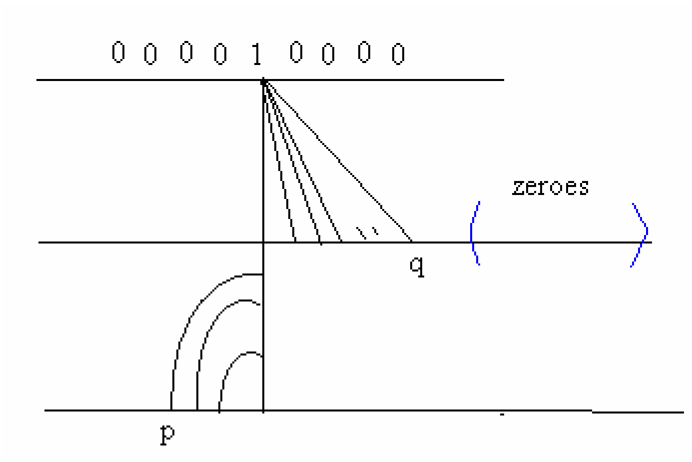
But in case of orthogonal bases there is no change in how the previous filter is used.



ARMA system identification:

There are two cases in this depending on the input.
If you can control the system input then we have three approximation techniques as follows:

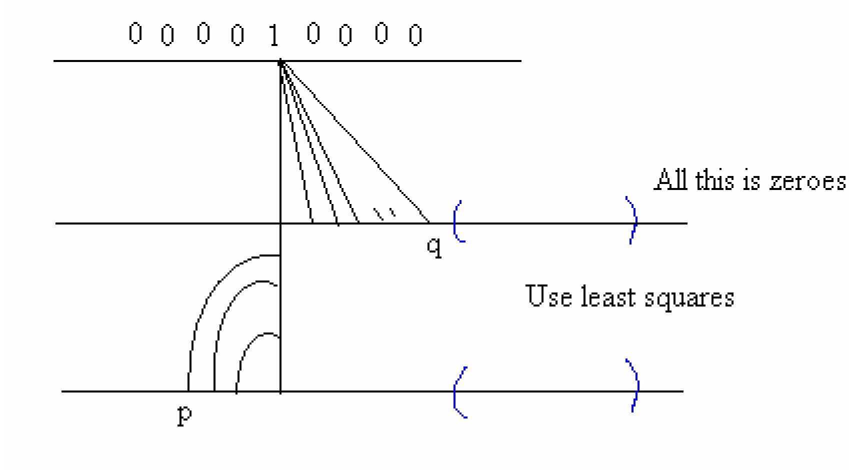
1. Pade Approximation Technique



In this technique we give an impulse as input. Assume that it has p poles and q zeroes.

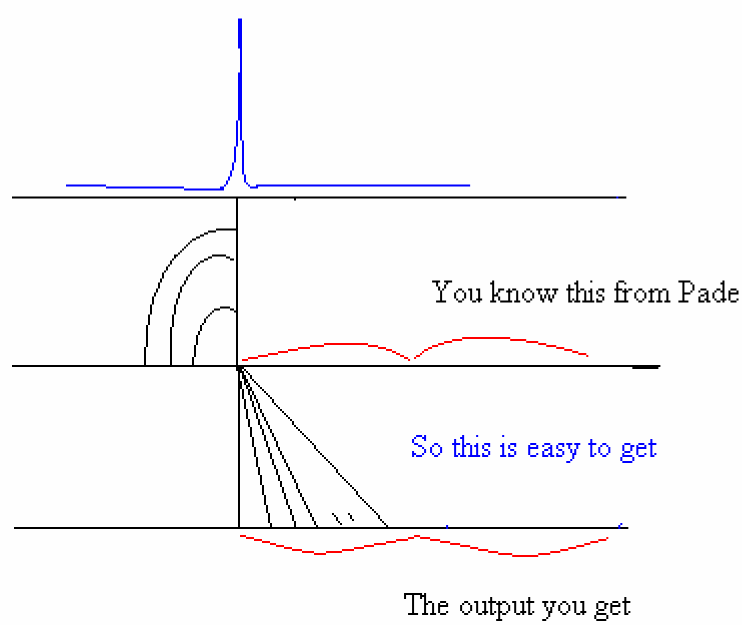
1. Since the input is finite, we know that the middle line will have all zeroes after the ‘q’.
2. And you know the p+q output after this middle input is given to the AR (the impulse response)
3. With this information (p+q output and q followed by 0 input) you can find out the coefficients of the AR filter.
4. Get the inverse of it, and then get the MA coefficients.

2. <u>Prony's Approximation Technique:</u>



In this you use the funda of all zeroes after q and Least Squares.
So consider lots of zeroes in the input and you get some huge output(for the AR filter).
Apply Least Squares and you can get the AR coefficients. Continue as above for MA.

3. <u>Shanks' Technique:</u>
Use the first part of Prony or Pade and get the AR coefficients. Now we are going to consider the equivalent MAAR



You have the impulse response of the AR system i.e the MA input. And given and impulse you know the system output. Use these to get the MA coefficients.

ARMA as a large MA

If you <u>can't control</u> the system input i.e cannot give an impulse then there is another technique

Model the ARMA as a large MA system. Give input in the order of millions, which will get you huge output. Now Least Squares using information will give you a nice approximation of the Impulse Response of the system.

In all the Prony, Pade and Shanks we first found the impulse response, which we now have. So you start with any of these techniques to get the remaining stuff.

Optimal ARMA Modeling

You can use a very good linear method.

We know our ARMA system looks like this

And the equation for a particular y_k will look like this

$$y_k = b_0x_k + b_1x_{k-1} + b_2x_{k-2}\ldots + a_1y_{k-1} + a_2y_{k-2} + \ldots$$

So basically you can have lots of such equations (for different values of k) which can be expressed as

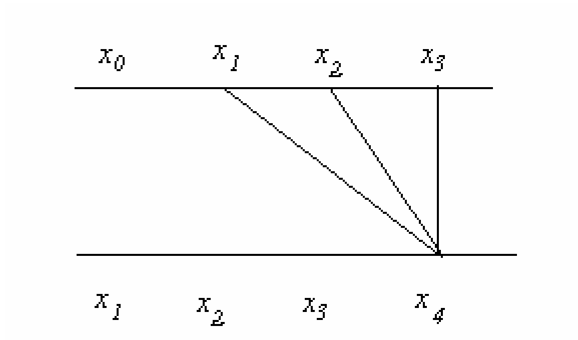
$$\begin{bmatrix} x_0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots \\ x_1 & x_0 & 0 & 0 & 0 & \ldots & y_0 & 0 & 0 & 0 & \ldots \\ x_2 & x_1 & x_0 & 0 & 0 & \ldots & y_1 & y_0 & 0 & 0 & \ldots \\ x_3 & x_2 & x_1 & x_0 & 0 & \ldots & y_2 & y_1 & y_0 & 0 & \ldots \\ x_4 & x_3 & x_2 & x_1 & x_0 & \ldots & y_3 & y_2 & y_1 & y_0 & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \\ . & . & . & . & . & \ldots & . & . & . & . & \ldots \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ .. \\ .. \\ a_1 \\ a_2 \\ a_3 \\ .. \\ .. \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ .. \\ .. \\ .. \\ .. \\ .. \\ .. \\ .. \\ .. \\ .. \end{bmatrix}$$

This again may look like the Toeplitz matrix but isn't, its more of double Toeplitz (notice the x's and y's). Solve this using Least Squares and you will get the linear, optimal ARMA system identification.

Applications of Least Squares:

1. Stock Market Prediction

   If the next day's stock market value is dependent on the rise or fall in the pass few days then you can model it as the following system, where all the x's are the stock market sensex everyday.



2. Innovations Process
3. Used for Compression