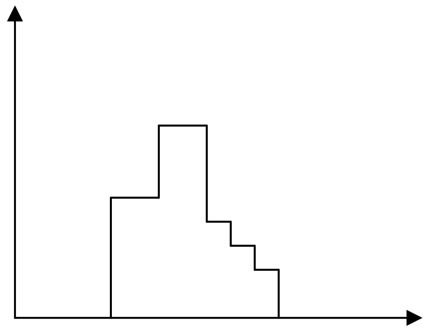
Filter Banks

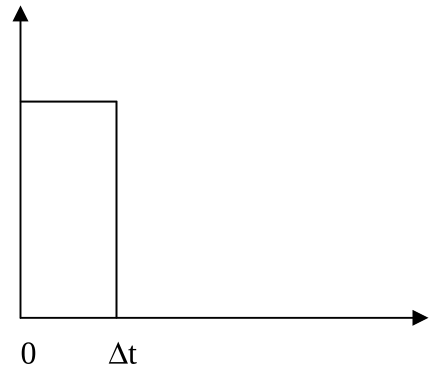
Before we do filter banks some other stuff.

Sampling and reconstruction principles are used for designing devices like speakers and soundcard. They do reconstruction upto 8khz

You can use a hold for reconstruction



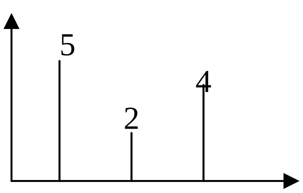
Time domain



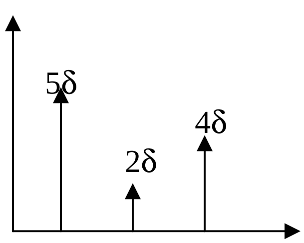
Frequency domain

Fig 1

To have this kind of a sample and hold effect
We need to first stretch and then convolve
Theoretical view:
If the signal is like below then the area under the curve is 0 as 5-2-4 have 0 area
This will result in 0 output



So what we need to do is to have unit area for 5-2-4.Hence convert them to dirac delta.

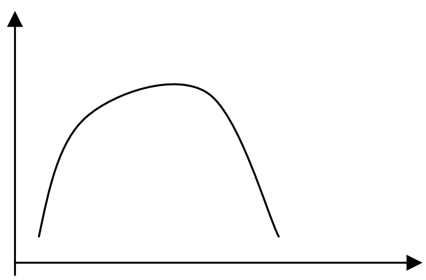


So the effect achieved as opposed to using just 5-2-4 are as follows

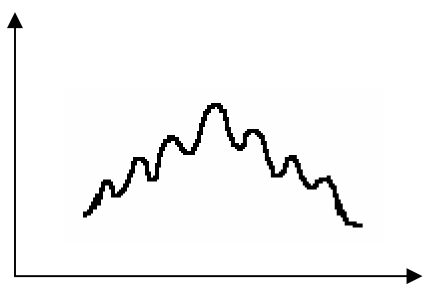
1. the first stretch caused one repeat in the $-\pi/2$ to $+\pi/2$ domain, but the dirac guys cause infinitely many repeats in the infinite freq domain
2. which will give you a fourier transform kind of a thing
3. so you have to use a low pass filter kernel to get the original freq response

Now the hold circuit shown in figure one is not so good to use
Mostly op-amps have this kind of a hold effect
So they use an analog lowpass filter after this

People who actually use this reconstruction technique are mainly LCD reconstruction and sometimes for huge T.V monitors
T.Vs ideally have to use the figure shown below for reconstruction ie this is how the phosphors have to get illuminated



But due to the R-G-B funda what they really do is use this for R-G-B each



and due to persistence of vision it actually adds up to the above

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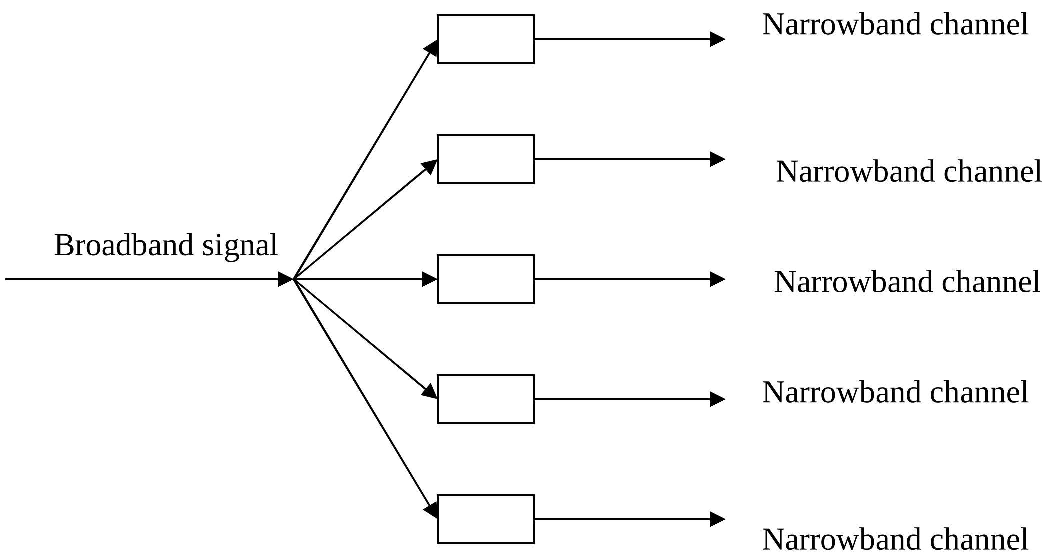
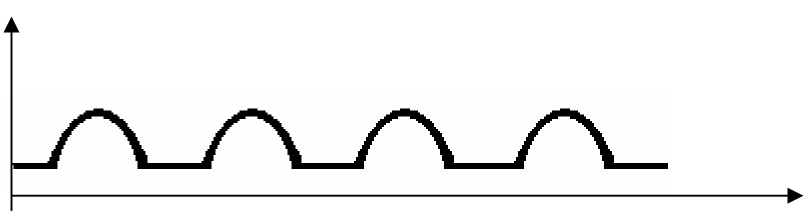
So coming back to what exactly is a **filter bank**

It looks something like this



Advantages:
This concept of filter banks is good for the following reasons
Since you can separate out the narrowband channels its good for analyzing or processing complex signals as you can definitely separate them out into many narrowband channels which are comparatively simple to process.

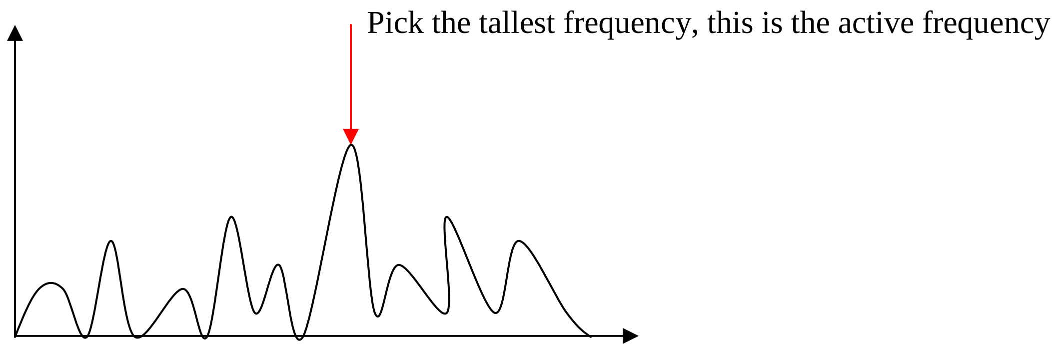
It can also be used for voice reconstruction or compression
Usually when you talk it's the air that passes the glottis, which produces some kind of vibrations in the air which look like this



The mouth cavity acts as a filter kernel for this signal.
This principal frequency and the overtones are actually your active frequencies.
Since you do windowing they appear as many.
These are the only frequencies that u have to actually care about
So what you can do is use a filter bank and separate out the active frequencies,

How do you find the active frequencies from the others, which are produced due to windowing

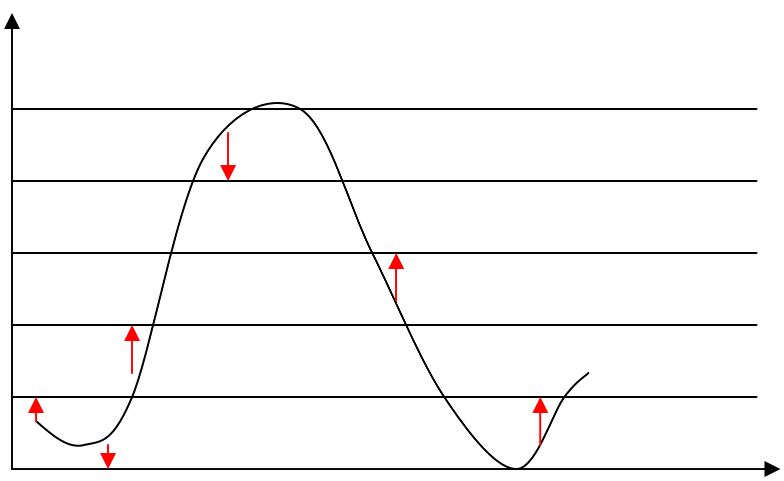
After the filtering you will get the frequency response that will look something like this



Compression

Using filter banks for compression of MP3 files

This is how quantization looks

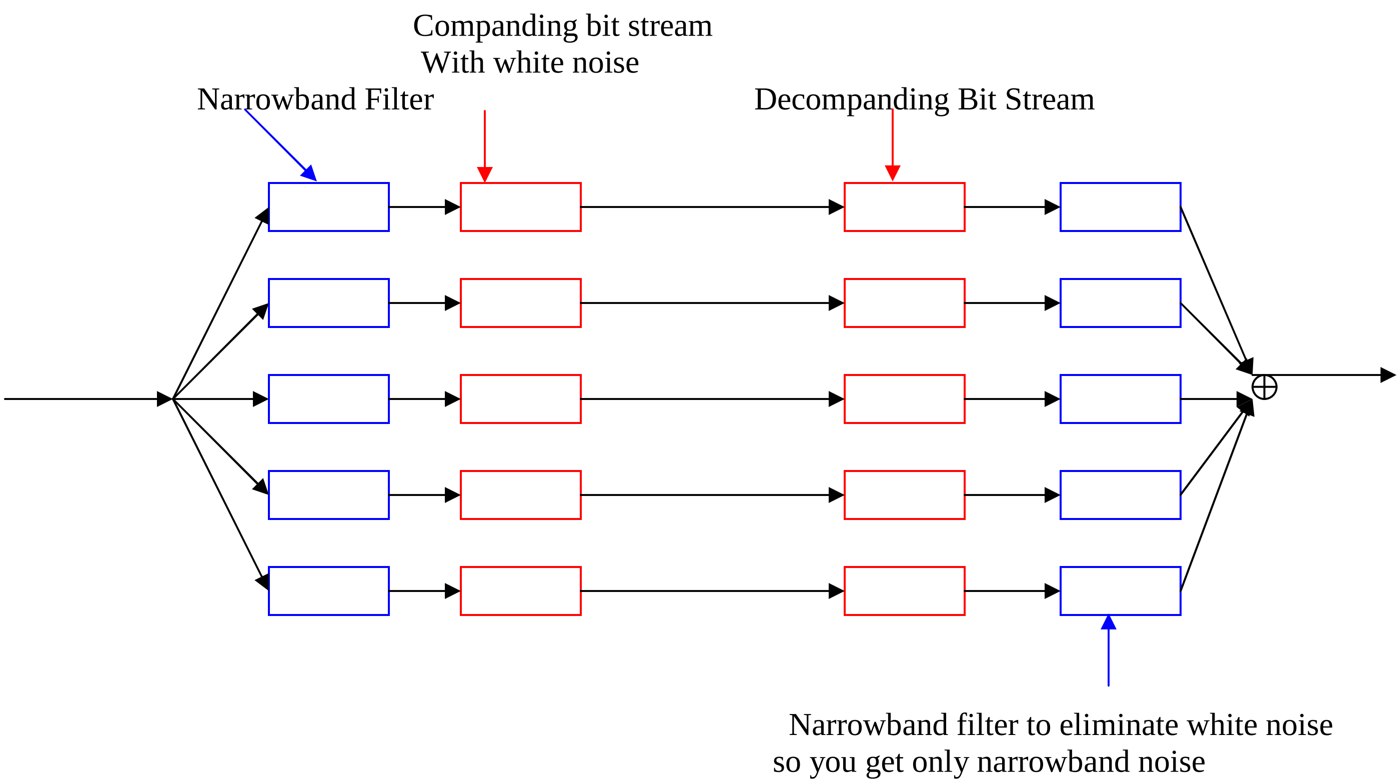


All the red arrows are the error or noise induced due to quantization. If the quantization levels are further apart (which means lower no. of bits), the noise will be more. But due to the persistence of hearing louder sound masks smaller sounds.

So if u are quantizing a loud sound u can use fewer levels, in spite of more quantization noise and thus have better compression.

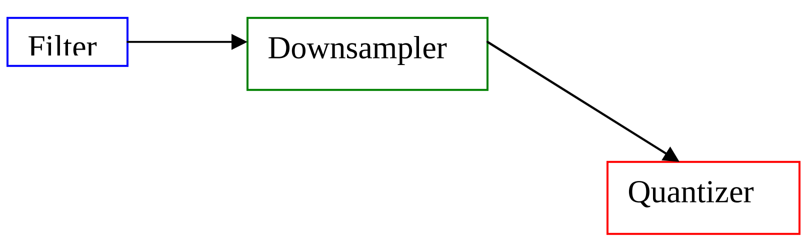
But in spite of the loud sound, you may not be able to mask all lower frequencies. Like an opera singer may not be able to mask the base voice of Amin Sayani.

So if you want to do some kind of compression you could use the following



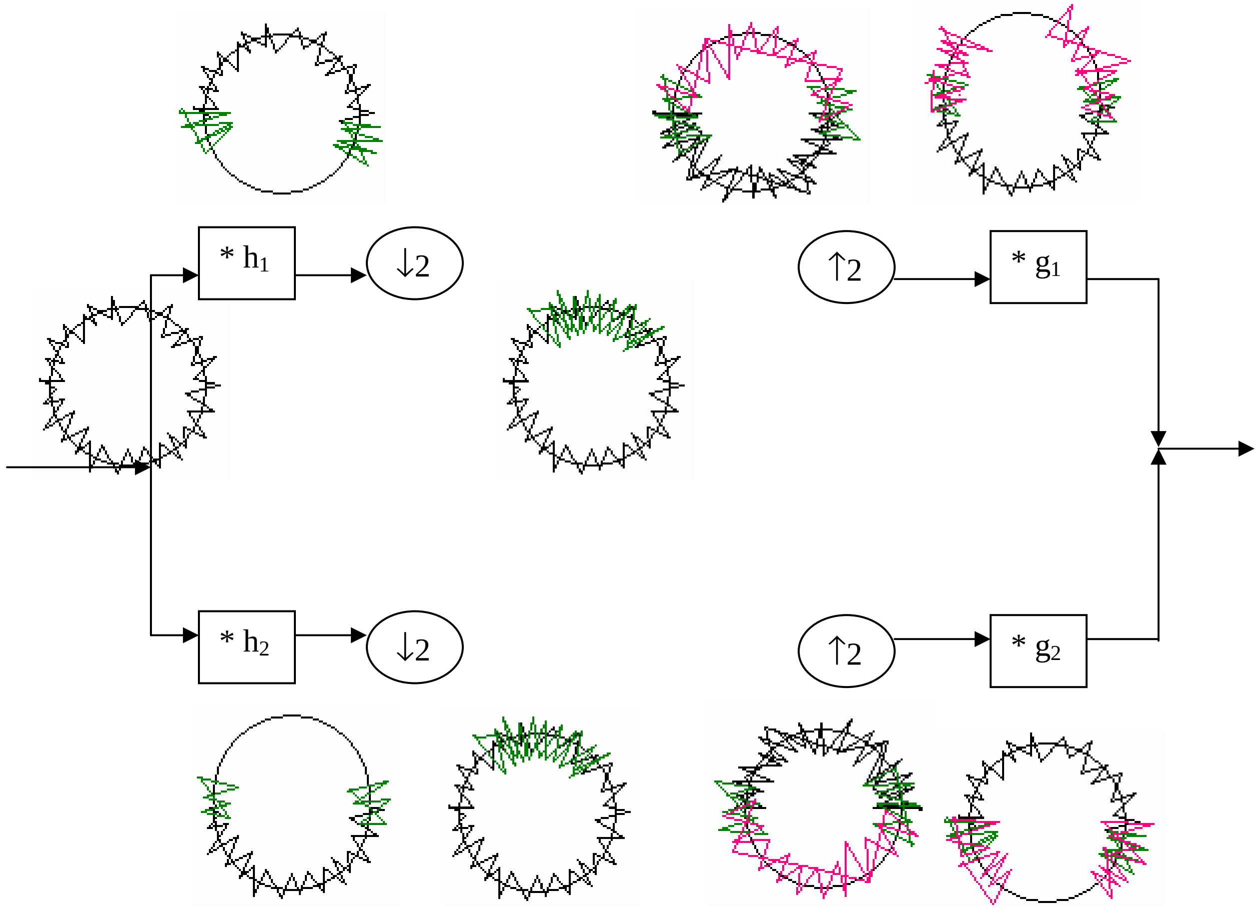
So this is how we are trying to achieve some compression.
But what we have ended up doing is just splitting the broadband signal into 12 narrowbands(in case of MP3 sound). This is not enough compression.

A better thing we could do is use a downsampler (where D = 12 in case of MP3)
Which would look like



Quadrature Mirror Filtering

Now lets take the example of a simple filter bank where we split the signal using a low-pass and high-pass filter

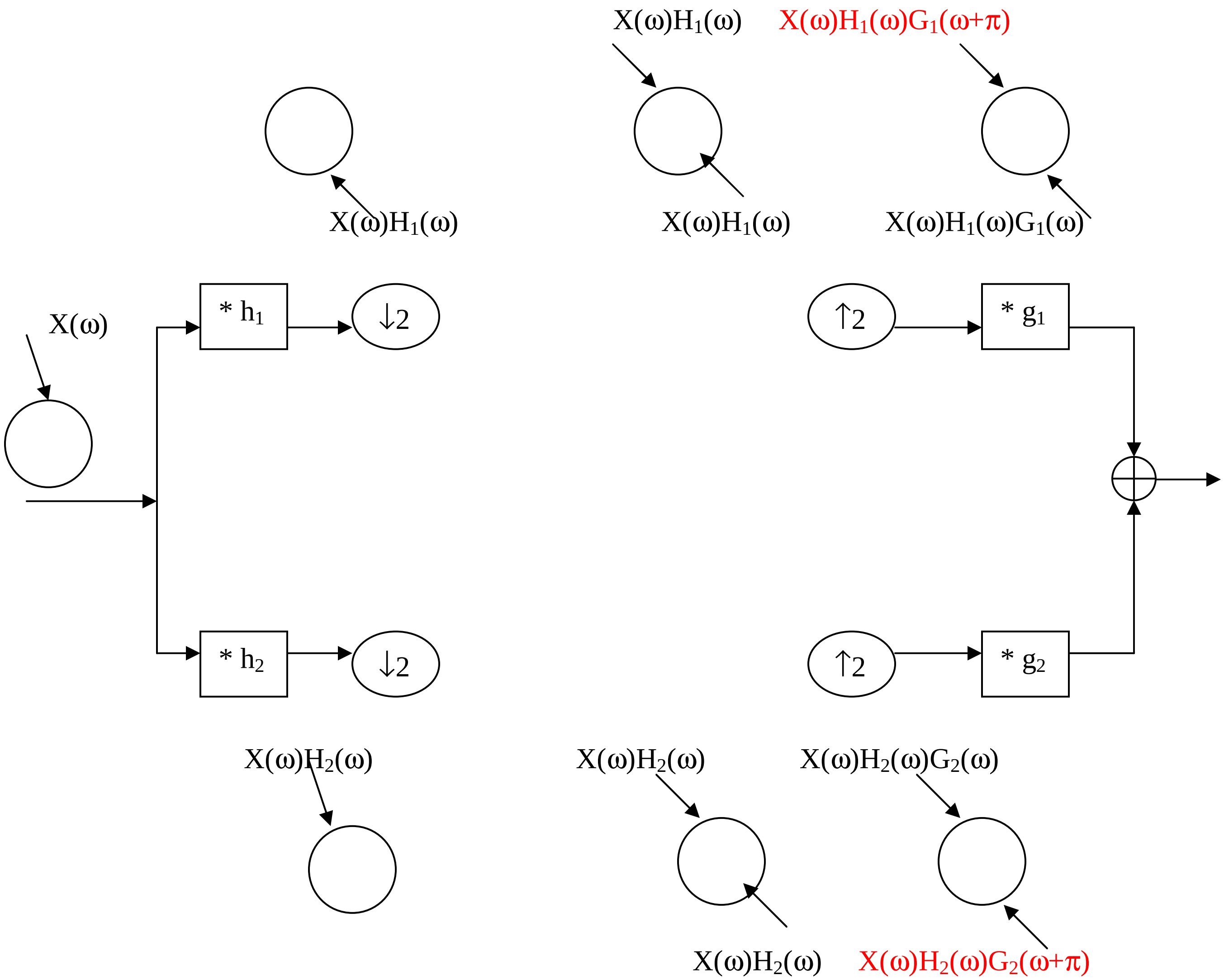


Now here we are really not concerned with what is happening in the middle layer (it could be a quantizer had we been thinking of compression, or anything)

Now initially the signal will get split into high and low frequencies. But as little of the higher frequencies pass through the low pass filter and the same for lower frequencies, there is a little smudging (indicated by the green part).

Next, both the downsamplers will cause a stretch in this smudged signal, to give the green part in the center.

Now we assume some process operating on this downsampled version of our signal, in the middle part, which will cause some smudging/distortion, and also after upsampling by 2, we will get a repeat in the signal spectrum. But here the green part, becomes a part of the spectrum, and hence its aliases will actually not be recognizable in the signal (although the drawing shows the different colour). The g filters will remove the repeat, and the final spectrum is obtained.



The figure above shows the frequency response after every step of the low pass and high pass filters.

The red marked areas are the effects of aliasing(the green part which we want to eliminate). Just by itself we may not be able to design g_1 and g_2 such that the green part gets eliminated in the output, its because once you reach the right hand side, you really cant differentiate between the smudging and the actual signal.
However you can make use of the fact that the green part is some high frequency component in the upper half and some low frequency component in the lower half. Hence we can use the initial high pass filter to remove the high pass component in the RHS, and the same applies for the low pass filter.

We know that the final addition is this,

$X(\omega)H_2(\omega)G_2(\omega) + X(\omega)H_1(\omega)G_1(\omega) + X(\omega)H_2(\omega)G_2(\omega+\pi) + X(\omega)H_1(\omega)G_1(\omega+\pi)$
Out of that we need

$X(\omega)H_2(\omega)G_2(\omega+\pi) + X(\omega)H_1(\omega)G_1(\omega+\pi) = 0$

Because we want to cancel out the aliasing,

$H_2(\omega)G_2(\omega+\pi) + H_1(\omega)G_1(\omega+\pi) = 0$

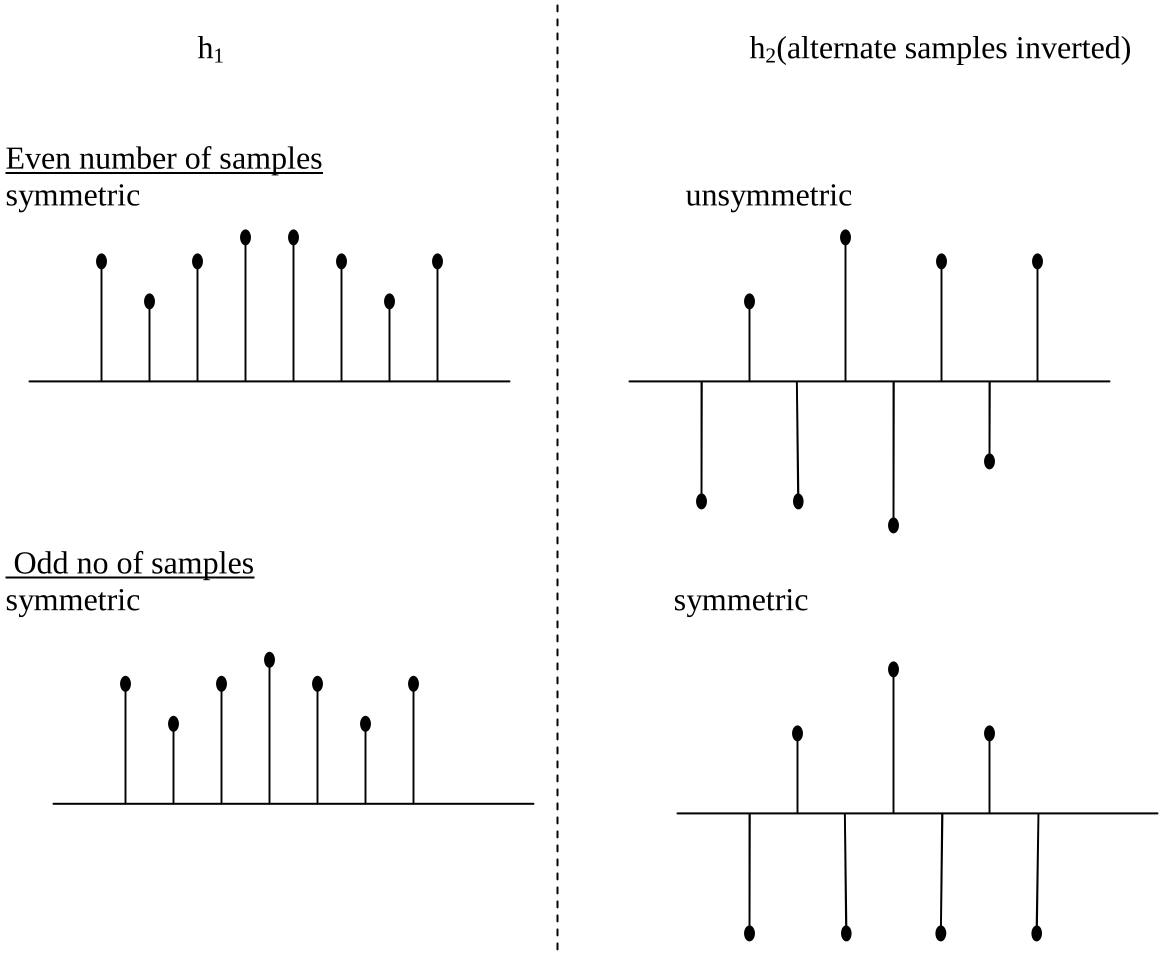
$G_1(\omega+\pi) = H_2(\omega)$
$G_2(\omega+\pi) = - H_1(\omega)$

This thing is called Quadrature Phase Mirroring

Now if we have the low pass filter having frequency response $H_1(\omega)$
We know that for the Perfect Alias Canceling high pass filter we need the frequency response to just be shifted by π.
We could write its frequency response as $H_1(\omega+\pi)$

Now a shift of π in the frequency domain is equivalent to multiplying by $e^{j\pi t}$ in the time domain.
Hence every alternate sample will get inverted (because $e^{j\pi t}$ will effectively do $(-1)^n$)
Therefore h_2 is same as h_1 except for the fact that h_2 has every next sample of h_1 inverted.
And since g_1 and g_2 also depend on the h_1 and h_2 we only need to design one filter h.

Each of these h_1,h_2,g_1 and g_2 need to be linear phase filters, because we cant afford to shift the phases of the narrowband signals in the filter banks, we finally have to synthesize them into one broadband again.
Therefore they have to be symmetric filters.



Therefore we need our linear phase filter h(based on which all h_1,h_2,g_1 and g_2 are designed) to have odd number of samples.

All this stuff combined together gives us our Quadrature Mirror filtering.