Power Spectrum Estimation

There are basics representations of signal which can be converted to other one in different ways.

Auto-correlation γ | Power density function Γ

a Coloring Filter coefficients | κ Reflection coefficients

Methods to find Time–Average auto-correlation function

Unbiased estimation method

Here we partly open the boxes and find the auto-correlation such that

$$E\,[X_nX_{n-i}] = \underset{n}{A}\; X_nX_{n-i}$$

$$= \frac{1}{N-i} \sum X_n X_{n-i} \qquad \text{n going from i to N}$$

Here variance is $E\,([1^{st}\text{ term} - 2^{nd}\text{ term}])^2$

Since there are few data points for larger lags we have more variance at larger lags.

Biased estimation method

$$E\,[X_nX_{n-i}] = \frac{N-i}{N}\, E\,[X_n X_{n-i}] \qquad \text{n going from i to N}$$

This is similar to windowing signal (auto-correlated) with a triangular window and doing average.

Less variance than unbiased estimate.

Relationship between Energy, Power spectrum and Periodogram.

Periodogram is Fourier transform of average (windowed) auto-correlation function.
Expected value of Periodogram is Power spectrum.

So by averaging Periodogram we get the Power Spectrum of signal.
Also in direct method by taking Fourier transform of auto-correlation function we can get the Power spectrum.

Auto-correlation gives the energy spectrum.

Non-parametric methods for Power Spectrum Estimation

1. Bartlett Method

In Bartlett method we divide the signal into blocks, find their periodograms and average them to get the Power spectrum. (The data segments are non-overlapping).
The final effect is true power spectrum convolved with a window.
Due to windowing (leakage frequency due to side lobes) the frequency resolution is low.

2.Welch Method

It is same method than above with some modifications –

I) Data segments can be overlapping.
II) Window the data (signal) before computing Periodogram (we may use different windows for each segment)

This method has got better precision but less frequency resolution than Bartlett method.

3. Blackman-Tukey Method

In this method we windowed the auto-correlation sequence and take Fourier transform to get power spectrum estimate (Periodogram) in effect we smooth out the Periodogram.
It has better variance (even at large lags) and better precision than above two methods.
But frequency resolution is less than the others.

Parametric method for Power Spectrum Estimation

Theme:
In these methods we assume that signal is output of a system having white noise as an input .
We model the system and get its parameters i.e. coloring filter coefficients and predict the power spectrum.

Yule-Walker method

We estimate the Auto-correlation. then we find the ‘a’ s coloring filter coefficients which are model parameters.
To find the ‘a’s we use Levinson-Durbin algorithm.

From these a’s we again find the γ and then Power spectrum.

Burg Method

We have seen the lattice filter equations for forward and backward prediction error filters.

$$Q_5(n) = Q_4(n) - \kappa_4 R_4(n-5)$$

$$R_5(n) = R_4(n-5) - \kappa_4 Q_4(n)$$

Q is the error quantity (least square).

So if we minimize the error by selecting κ we can model the 'a's

For that we predict the κ_1 and using this we find other reflection coefficients using same lattice structure.
Using Levinson-Durbin algorithm we model the system and find the 'a's from which we can get the power spectrum estimate.

Predict : κ_1 =(autocorrelation of x(n))/(energy in x(n))

And we can find the

$$\kappa_4 = \frac{-\langle R_4 \text{ delay } 5, Q_4\rangle}{\langle R_4, R_4\rangle}$$

Using property that forward and backward coefficients are the same one

$$\kappa_4 = \frac{-\langle R_4 \text{ delay } 5, Q_4\rangle - \langle Q_4, R_4 \text{ delay } 5\rangle}{\langle R_4, R_4\rangle + \langle Q_4, Q_4\rangle}$$

Thus we can find R1 and Q1 from K1…then K2 from R1 and Q1 …then K2 and so on…

From κ's we find a's and the Power spectrum.
This method has higher frequency resolution and computationally efficient.