A bit of history -

Hipparcus, while trying to model the orbits, took the help of spirals. The reason he could do it was - spirals can model anything.

Properties of spirals -

1. They are linear and time invariant.
2. Any signal can be modeled using them.
3. They are orthogonal to each other.

Linear systems are important because of apparently a circular reason -

Since they are easy to analyze, we try to find many applications that are linear. Similarly, because there are many applications, the theory is so important.

The "differentiator" -

The Discrete domain has a "Delay" operator. The Analog domain doesn't, because it is hard to define "unit time". Thus, we have the "differentiator" in Analog domain. The frequency response is y=w, where w is the angular frequency of the signal (we call it D1).

Why we analyze the differentiator -

The differentiator itself is a linear system. Also, the derivative of x(t) is always 90 degrees leading to the x(t) in phase, and the magnitude is proportional to the magnitude of the angular velocity of x(t).

The problem -

The differentiator loses D.C. So, to get some output at low frequencies, we use x(t)+x'(t).(we call it D2) The corresponding curve is $y=\text{sqrt}(1+w^2)$. Till corner frequency $w_c=1$, the factor 1 dominates, after that w^2 dominates. This curve asymptotically becomes x'(t).

The inverse -

The inverse of the differentiator is the integrator, with curve y=1/w. (rectangular hyperbola, and we call it I1), and that of the system x(t)+x'(t) is the curve $y = 1/\text{sqrt}(1+w^2)$, which asymptotically meets the x-axis and the integrator I1. The corner frequency is still 1. This is nothing but First order low pass butterworth filter (we call it I2).

How to use the prototypes –

To construct,

HPF - Cascade 1st order LP butterworth filter(I2) & the differentiator D1.

Second order Low pass butterworth filter - Add one more differentiator to D2, to get the output as x(t)+x'(t)+x''(t).

Higher order filters - Cascade suitable low order filters.

Band pass filter - Cascade an HPF and an LPF.

Band reject filter - Cascade an LPF and an HPF.

Notch filter - Cascade an HPF and an LPF with the cut off frequencies close to each other.

