FIR Filter Design

Steps involved in FIR Design:

1. Decide the required response in frequency domain with zero phase.
2. By IDTFT find out the signal in time domain considering zero phase in frequency domain. The IDTFT operation can be viewed as calculating Fourier Coefficients of the signal in frequency domain. This is DUALITY OF FOURIER ANALYSIS.
3. The time domain signal is the filter kernel (impulse response). Do convolution to get output. As the impulse response is infinite and non causal, it cannot be used directly.
4. Multiply it by a window function to make it finite. This spreads out the response in the frequency domain and causes ringing. (Duality of Fourier Transform- Multiplication in time is convolution in frequency)
5. To make it causal delay the time response by 'k' samples. The frequency response gets a phase lag of 'kw' where 'w' is the frequency of spiral. Hence the filters are linear phase.
6. For symmetric impulse responses (even responses), the imaginary parts of the spirals exactly cancel out giving only a real frequency response. For anti symmetric impulse responses (odd responses), the real parts exactly cancel each other giving only imaginary frequency response or equivalently a phase shift of 90^o

Window Functions:

There are many window functions used to optimize the impulse response and reduce ringing in the frequency response of the filter. The tradeoffs are the transition band width and the stop band responses.

IIR Filter Design

Why IIR Filters?

1. Easier to implement (lesser coefficients)
2. Effective usage of advanced analog filter design theory

Steps in Design (Bilinear Transform):

In Digital IIR filters, analog filters are imitated by use of different techniques. To do that we first model the analog "Differentiator" as digital filters.

1. The most basic model in the differencer. The differencer gives a phase difference of (90^o – w/2) instead of the usual 90^o for each spiral of 'w' frequency.
2. To get the exact differentiator use the corrected derivative of the previous sample of the same spiral. The sum of the error with present value leads to exact output phase of 90^o as needed. This is modeled using an auto regressive filter in cascade with the differencer
3. But, the magnitude of such a cascade would imitate a differentiator of different analog frequency. The entire analog frequency range from $-\infty$ to ∞ is translated to digital frequency domain $-\pi$ to $+\pi$.
4. This is called frequency warping. The lower frequencies are not affected much but the higher frequencies are transformed by the warping function.
5. Thus each differentiator in an analog filter can be replaced with this cascaded filter and the entire filter can then be simplified to get the final filter.

The model for Differentiator (Bilinear Transform)