

Introduction

A resistor network can be modeled by using simple linear equations. Further, a network of inductors and capacitors can be modeled into linear differential equations. These linear differential equations can be further modeled into blocks of differentiator and inverse differentiator, which are filter models.

Interesting history

Oliver Heaviside discovered operational calculus a tool for solving linear differential equations with constant coefficients. It happens that the operational calculus is very similar to Laplace transform procedures.

Operational calculus is about replacing the differential term by a variable (he replaced it by p) and then solving the equations applying normal rules of algebra. Whereas, the Laplace transform replaces one function F(t) of t by another f(s) of the new variable s by the rule: $f(s) = \int_{(0,\infty)} e^{-s*t} *F(t)*dt$.

An easy integration by parts gives the transform of the derivative of F(t): $L[F'(t)] = s*f(s) - F(0)$. If we replace s by p, and $F(0) = 0$, we recover Heaviside's expression.

OP AMPS

Consider an ideal stick balancing Gorilla. So if we move the stick (provide stimulus to the stick) gorilla will move so as to balance the stick. If we attach any load to the gorilla, the gorilla will move the load in accordance with stimulus given to the stick. So important to note that all the power to move the load is supplied by the gorilla and not the input. Thus we have modeled an ideal linear amplifier.

To device a linear amplifier is difficult, transistors are non-linear. Instead we design an amplifier with feedback and gain very high so the resultant gain depends on the feedback factor.

Gain with feedback is given by $A/(1+AB)$ where A is gain without feedback and B is feedback factor. If A is very high then resultant gain is $1/B$.

So the op amp is differential input amplifier, which does the work to keep the two input terminals shorted (thus initially when there is some difference it drops the output so as to maintain the input terminal shorted).

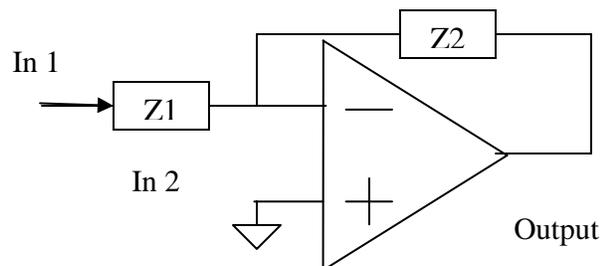
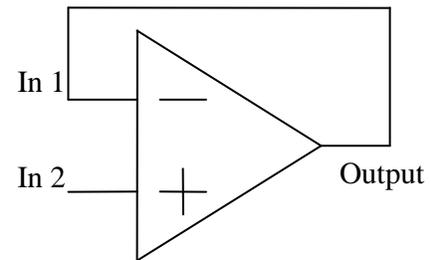
Virtual short – The op amp does as much work required to keep the two terminals short. Thus these two terminals are said to be virtually shorted.

Operational range – It is the range between which the output may vary (max supply range).

FAQ on op-amps – If the two terminals are at same potential then why there is an output?

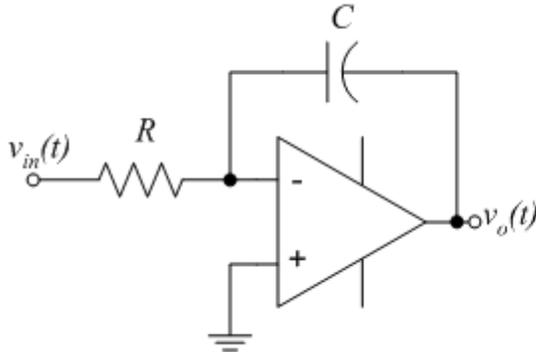
Answer – When the two terminals are shorted the short is with respect to the input voltages but actually there is some differential voltage (say k) that acts upon the amplifier and we get the output equal to $k*A$.

One more way to explain in terms of electrons – If $Z1 = Z2 = 1$ ohm resistance then the op-amp drops the voltage at output to make the electrons flow through resistance Z2 and so as to keep the condition of virtual short and it would follow this even if there is some load at the output. Thus relating it to stick balancing gorilla op-amp takes the entire load and is power supplier.



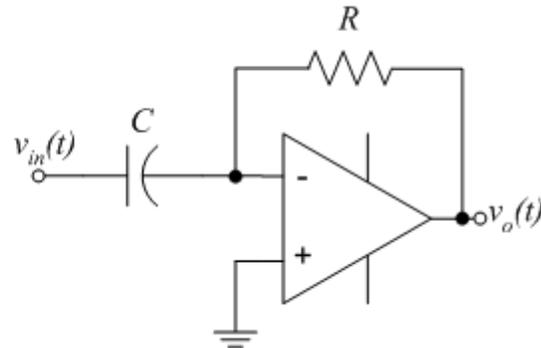
Some common and popular circuits using op-amps are –

1. Voltage Follower
2. I to V and V to I converter
3. Differentiator and integrator



Integrator

$$V_o(t) / V_{in}(t) = -1/j\omega RC$$



Differentiator

$$V_o(t) / V_{in}(t) = -j\omega RC$$

So if we can model differentiator and inverse differentiator using op-amps then we can model any complex transfer function. Thus we can model filters using op-amps.

Few points regarding op-amps having complex numbers in its transfer function.

The complex number there involves the term $j\omega$ it just means the transfer function at one particular frequency and for one particular spiral. But as we know that all these are LTI systems applying superposition we can find for the signal(decomposed into spirals).